4th ASLAN ROCK MECHANICS SUMPOSIUM

Overview of Some Problems of Rock Dynamics

Dr. Wang Mingyang

The Nanjing Engineering Institute

Contents

1. Research Background

1.1 Local damage effects of penetration

Penetration, perforation and spallation

1.2 Local damage effects of explosion

Influence of cavity size, damage zone size, free surface etc.

1.3 Meaning

Design calculation of protective structure

High Efficient Damage assessment of weapons

1. Research Background

1.4 Present state:

Empirical formula: based on energy and similarity principle, it is rough with large error of order.

Theoretical research: description of media behavior and its state variation is lack of accuracy and physical basis.

Numerical simulation: information sources of parameters, reliability of model are suspectable.

2. Fundamental Problems

2.1 General features of deformation and damage in rock

（1）conservation law

$$\int_S \vec{\sigma}_n \cdot \vec{u}\, dS + \int_V \vec{F} \cdot \vec{u}\, dV = 2\int_V W dV$$

W consists of:

(1) storage potential energy—volume characteristic

(2) internal dissipation energy（unrecovered deformation, cracks and slip line）

(3) motion energy（unsteady state and steady state）--volume characteristic

volumetric stress --satisfying momentum conservation

deformation--- satisfying mass conservation

2.1 General features of deformation and damage in rock

（2）stress characteristic

$$\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} = T\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} + \sigma_n \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + T\begin{bmatrix} 0 & 0 & 0 \\ 0 & \mu_\sigma & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$T = \frac{\sigma_1 - \sigma_3}{2}$$ **Maximum shear stress**

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2}$$ **Hydrostatic pressure**

2.1 General features of deformation and damage in rock

$$\mu_\sigma = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3} - -\sigma_2$$ **Influence on strength**

$$\mu_\sigma = \frac{T_{23} - T_{12}}{T}$$

$$T_{12} = \frac{\sigma_1 - \sigma_2}{2} \qquad T_{23} = \frac{\sigma_2 - \sigma_3}{2}$$

2.1 General features of deformation and damage in rock

（3）strain characteristic

$$\gamma = \varepsilon_1 - \varepsilon_3 \text{ -- maximum shear strain}$$

$$\varepsilon_V = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \text{ -- volumetric strain}$$

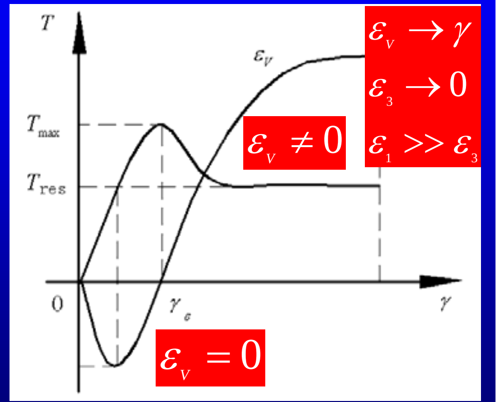
$$\varepsilon_V = \varepsilon_1 + 2\varepsilon_3 \text{ -- spherical symmetry} (\varepsilon_2 = \varepsilon_3)$$

$$\varepsilon_V = \varepsilon_1 + \varepsilon_3 \text{ -- plane strain } or \text{ cylindrical axial symmetry} (\varepsilon_2 = 0)$$

2.1 General features of deformation and damage in rock

（4）stress-strain

1) **Elastic state---continuous medium**
- **internal friction state----local shear state after peak value---quasi-continuous medium**
- **Mass movement---non-continuous medium**

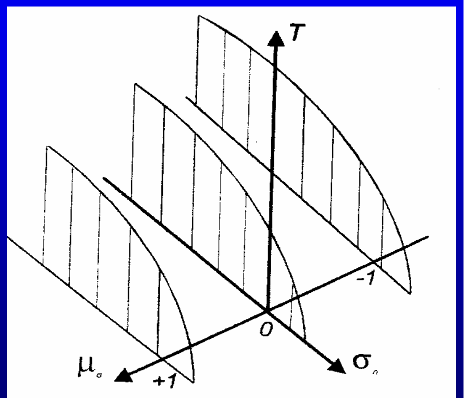


2.1 General features of deformation and damage in rock

（5）strength criteria

$$\tau_n = f(\sigma_n, \mu_\sigma)$$

Combination characteristic of new invariants---local plastic theory.



2.1 General features of deformation and damage in rock

- **Strength criteria indicates：when rock mass is subjected to shear, friction force increases and bonding force reduces.**
- **With increase of hydrostatic pressure on the shear front, the limit value of shear strength is also increasing, it reflects the appearance of friction force, either protecting rock mass from decomposition into various section，or reflecting a additional strength.**
- **Therefore，the shape variation of rock mass is mainly related to elastic, viscous and friction properties, but volume change is related to expansion and size of destroyed rock mass。**

2.2 Local-restrained deformation state in rock

A. elastic state

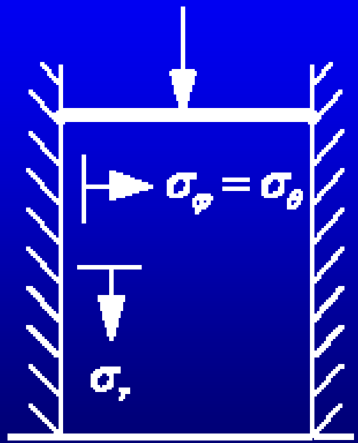
$$\frac{\sigma_\theta}{\sigma_r} = \alpha \qquad \alpha = \frac{\nu}{1-\nu}$$

B. internal friction state

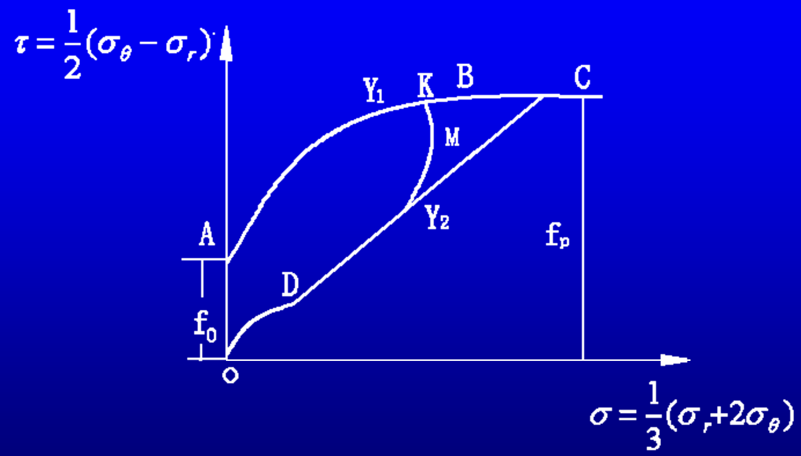
$$\frac{\sigma_\theta}{\sigma_r} = \alpha^* < 1 \qquad \alpha^* = \frac{1-\sin\varphi}{1+\sin\varphi}$$

C. hydrodynamic state

$$\frac{\sigma_\theta}{\sigma_r} = \alpha^* \approx 1$$



2.2 Local-restrained deformation state in rock



A—B—C states reflect relative value of friction force separately.

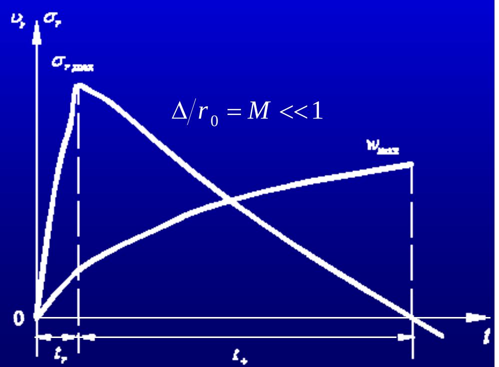
2.3 Deformation state of penetration and close-in explosion

$\frac{t_r}{t_+} = 0$ **on shock wave front** $\varepsilon_\theta = 0$

$\frac{t_r}{t_+} \approx 0.1$ **strain of stress wave** $\varepsilon_\theta \neq 0$

High stress and velocity changed rapidly only in certain narrow zone.

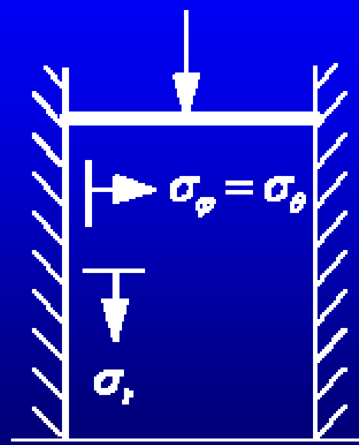
$$\varepsilon_r \propto \frac{\partial \varepsilon_\theta}{\partial r}$$



2.3 Deformation state of penetration and close-in explosion

$$\varepsilon_r \approx \varepsilon_v >> \varepsilon_\theta = \varepsilon_\varphi$$

The deformation state in close-in is close to one-dimensional strain state.



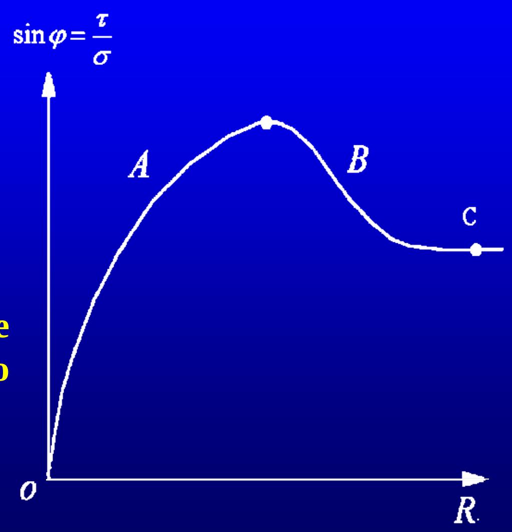
2.4 Stress state of penetration and close-in explosion

$$\upsilon \to C_p, \alpha^* \to 1$$

$$K\varepsilon >> \tau_0$$

$$\sigma_r = \sigma_\varphi = \sigma_\theta = -p$$

The deformation state in close-in is close to hydrodynamic state.



2.4 Stress state of penetration and close-in explosion

Ignoring that the strength is not depend on "strength extinction", the energy consumed to volume change is large more than that consumed to overcome the resistance of shape variation.

2.5 Energy distribution of penetration and close-in explosion

The main portion of consumed energy is related to different range: volume variation which includes heating and qualitative change. It is proportional to the cube of cavity radius r_c^3, but the shear stress on the sliding surface and damage (forming a new surface) is proportional to the square of cavity radius r_c^2.

$$\frac{\text{damage energy stage}}{\text{motion energy stage}} = \frac{\sigma\varepsilon}{\sigma} = \varepsilon \approx 0.001---0.005$$

The motion energy is the main consumed energy in close-in zone for restricted friction heating.

2.6 Velocity field of penetration and close-in explosion

（1）sound velocity of dynamic stress state in media

A. elastic state

$$C_P^A = \sqrt{C_V^2 + \frac{4}{3}C_S^2}$$

B. internal friction state (complete plastic or local plastic)

$$\xi^* = \frac{\nu}{1-\nu} \qquad C_P^A = C_P^B$$

C. hydrodynamic state

$$\xi^* = 1 \qquad C_V^C = \sqrt{\frac{\lambda + 2\mu/3}{\rho_0}} = \sqrt{\frac{K}{\rho_0}}$$

2.6 Velocity field of penetration and close-in explosion

（1）sound velocity of dynamic stress state in media

There are two main characteristics reflected by sound velocity value: the ability of solid body to resist volume compression and shearing deformation. But one sound velocity value cannot include the characteristic of elastic or non-elastic stress state at the same time.

2.6 Velocity field of penetration and close-in explosion

（2）particle kinematic velocity of dynamic stress state in media

The stress waves represent the section with most high velocity which reflect the compressibility and irreversible deformation. the action index of dynamic stress is the particle kinematic velocity. The transport carrier of energy and momentum is the deformation and motion of particles.

$$\varepsilon_r \approx \varepsilon_v >> \varepsilon_\theta = \varepsilon_\varphi$$

2.6 Velocity field of penetration and close-in explosion

（2）particle kinematic velocity of dynamic stress state in media

Based on shear and expansion equation:

$$\upsilon(r,t) = \dot{a}\left(\frac{a}{r}\right)^k$$

The close-in media is with incompressible relations:

$$\varepsilon_r + k\varepsilon_\theta = 0$$

$$\varepsilon_r \propto \varepsilon_\theta$$

2.6 Velocity field of penetration and close-in explosion

（2）particle kinematic velocity of dynamic stress state in media

The error of local damage effects caused by penetration and close-in explosion is mainly attributed to description of velocity field with large error of order.

2.6 Velocity field of penetration and close-in explosion

（2）particle kinematic velocity of dynamic stress state in media

Based on mass conservation:

$$\frac{1}{3}\frac{\partial}{\partial r_0}(r_0+w)^3=\frac{\rho_0}{\rho}r_0^2 \qquad r=r_0+w(r_0,t)$$

$$w(r)=r-\sqrt[3]{r^3-a^3}$$

$$\upsilon_r=C_p\varepsilon_r=C_p\left(1-\left(\frac{1}{\sqrt[3]{1-r_*^3}}\right)^2\right)$$

$$\approx \eta C_p\left(\frac{a}{r}\right)^k=C_p^*\left(\frac{a}{r}\right)^k$$

2.6 Velocity field of penetration and close-in explosion

（2）particle kinematic velocity of dynamic stress state in media

Under the limited deformation state in close-in zone, the stress state can be determined by momentum conservation equation, and the velocity field can be determined by the law of mass conservation。

3. Some New Development

3.1 Penetration Effects in Rock

3.2 Explosion Effects in Rock

3.3 Dynamic Nonlinear Problems in Rock

3.1 Penetration Effects in Rock

（1）similarity relation of penetration

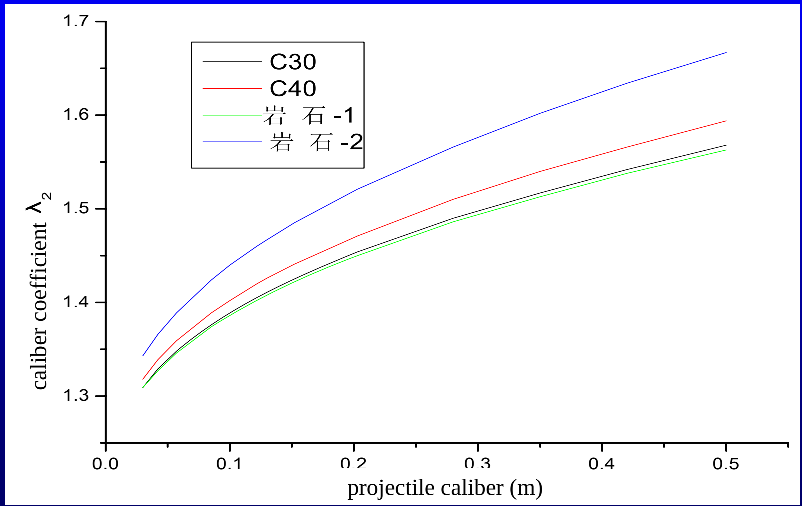
$$\frac{R_*}{a}=\eta_1\sqrt{\frac{a}{l}}=\eta_1\chi^2$$

$$\frac{a}{b}=\eta_2\chi$$

This reveals the physical and geometrical conditions of perforation formation along with the ratiometric conversion relations, when projectiles with different caliber penetrate the same media.

3.1 Penetration Effects in Rock

（2）scale effect of penetration experiments



3.1 Penetration Effects in Rock

（3）calculation of normal penetration depth

if $\upsilon_0 \leqslant 1000$m/s, penetration depth of projectile:

$$h=\frac{m}{B}[\upsilon_0-\frac{A}{B}\ln(1+\frac{B}{A}\upsilon_0)]$$

if $\upsilon_0 > 500$m/s, influence on number item is less than5%, penetration depth of projectile:

$$h=\frac{M}{d^2}\lambda_1\lambda_2K_p\upsilon_0$$

where $\lambda_1=\frac{ctg\ \beta_0}{\kappa_0\pi}$ $\lambda_2=2/(1-\frac{8}{9}\eta_2\chi)$ $K_q=\frac{1}{\rho_0 c_p}$

3.1 Penetration Effects in Rock

（4）calculation of penetration into mutiply- media

$$h=\sum_{k=1}^{n}H_k+h_{q(k+1)}-\sum_{k=1}^{n}H_k\frac{K'_{q(n+1)}}{K'_{qk}}$$

Equivalent principle of wave impedance in different media layers

3.1 Penetration Effects in Rock

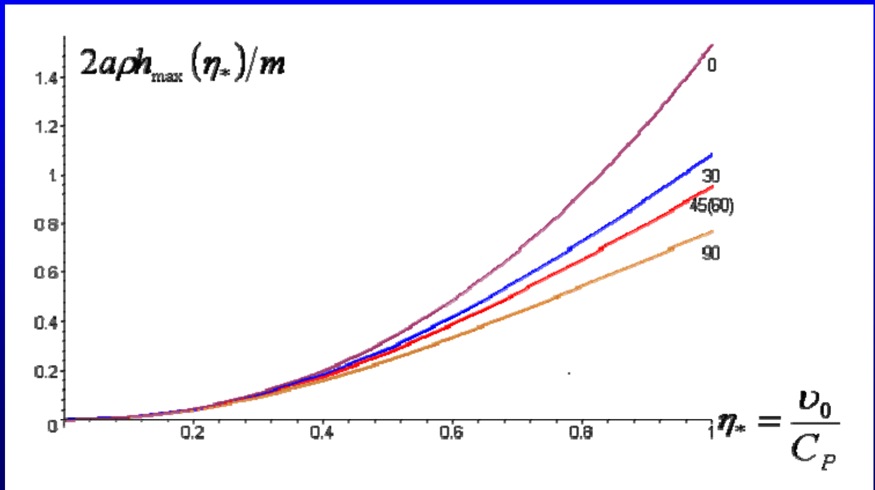
（5） oblique penetration calculation

$$h=\frac{M}{d^2}\lambda_1\lambda_2K_p\upsilon_0\cos(\varphi_0)$$

In process of oblique penetration, the total amount of volume variation of media is increasing, which is equivalent to decrease of velocity.

3.1 Penetration Effects in Rock

（6） high velocity penetration –rigid projectile



3.1 Penetration Effects in Rock

（7）critical condition of perforation

$$R^{*} = \eta_{1} r_{0} \sqrt{\frac{r_{0}}{l}}$$

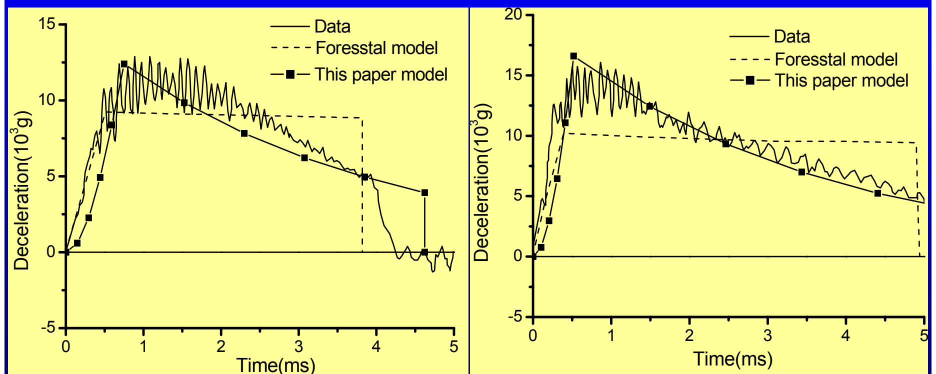
3.1 Penetration Effects in Rock

（8）perforation coefficient

$$\zeta = 1 + \frac{R^{*}}{h_{0}}$$

3.1 Penetration Effects in Rock

（9）overload problem in process of penetration and perforation



3.2 Explosion Effects in Rock

（1）similarity relation of explosion

$$\frac{R_d}{r_c}=\left(\frac{\rho C_p^2}{4\sigma_c}\right)$$

The cavity radius is accordant with the principle of geometric similarity, the radius of nonelastic deformation zone is proportional to the cavity radius.

3.2 Explosion Effects in Rock

（2）explosion compression, damage radius and base period

$$r_c=\frac{\alpha Q^{1/3}}{\left(\rho C_p^2\sigma_c^2\right)^{1/9}}$$

Propagating period of waves in elastic zone is proportional to that in nonelastic zone.

$$T=2R_d/C_p$$

The space of damage process and characteristic parameter of time are highly depended on media behavior in close-in zone. The existence of plastic zone close to cavity is equivalent to size increase of radiation source of elastic waves.

3.2 Explosion Effects in Rock

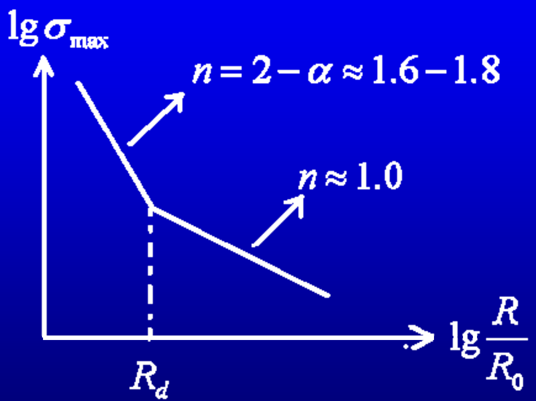
（3）attenuation regularity in close-in zone (spherical symmetry)

internal friction state in close-in zone

$$\sigma_r\sim\frac{1}{r^{2-\alpha}}$$

elastic state in far zone

$$\sigma_r\sim\frac{1}{r}$$

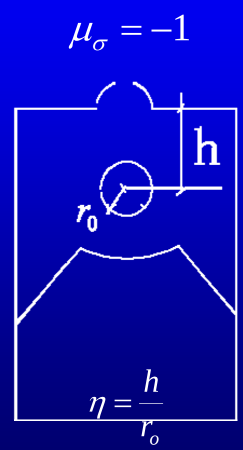


The main reason of attenuation is friction and variation of media state, the track of attenuation is damage zones。

3.2 Explosion Effects in Rock

（4）influence of free surface on explosion effects

The existence of free surface can lead to dissymmetry of damage zones. If the free surface exist, there are surface damage zone and scabbing damage, besides central damage zone.
Only if relative depth of burial η>3, the central damage zone and scabbing damage zone appeared. If η＝3, the zone between projectile and free surface in not enough to form the central damage zone, because here the scabbing damage has reached the explosion cavity.



3.2 Explosion Effects in Rock

（4）influence of free surface on explosion effects

Experiment results indicate:

- With the existence of free surface, the attenuation regularity of maximum displacement and positive-phase duration of compression waves changed with the distance to explosion center is similar to that regularity of underground confined explosion.
- The influence of free surface on duration of waves is more than that of particle velocity.
- This is the reason that buried explosions with different depths can not be concluded in one similarity group only based on one explosion source parameter, but the explosion with the same buried depth and different charges is similar to each other.

3.2 Explosion Effects in Rock

（4）influence of free surface on explosion effects

For the damage effects caused by different buried depths, it can be determined by the filling coefficient in the specifications. Compared with surface explosion, the compression radius and damage radius subjected to completely confined explosion is 1.65 times higher than surface explosion.

$$n = 2 - \alpha \approx 1.65$$

3.2 Explosion Effects in Rock

（4）influence of free surface on explosion effects

Being similar to the calculation of compression and damage radius, the spalling radius is also determined by introduction of the filling coefficient.

3.3 Dynamic Nonlinear Problems in Rock

（1）shock test apparatus of rock mass

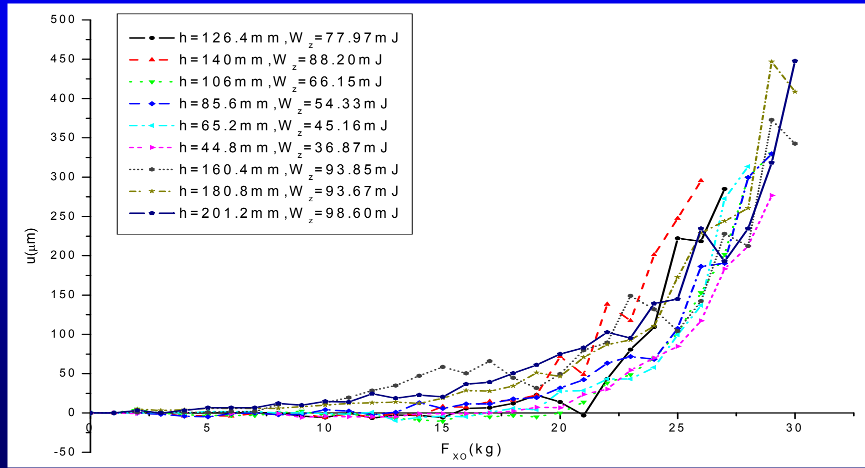
development of multifunction test system of dynamic characteristic: loading device and measurement system。

3.3 Dynamic Nonlinear Problems in Rock

work condition 1: only under action of vertical impact force W_Z.
work condition 2: under combined action of vertical impact force W_Z and horizontal static Fx.

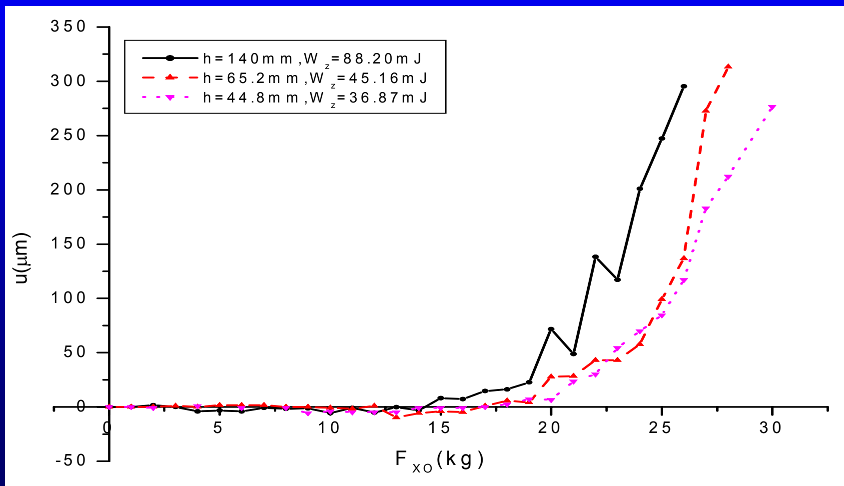
3.3 Dynamic Nonlinear Problems in Rock

(2) horizontal force Fx-u relationship



3.3 Dynamic Nonlinear Problems in Rock

(2) horizontal force Fx-u relationship



3.3 Dynamic Nonlinear Problems in Rock

(3) phenomena of ultra-low friction

F_0 / F_{X0}

No.	1	2	3	4	5	6	7	8	9
height of drop hammer (mm)	44.8	65.2	85.6	106.0	126.4	140.0	160.4	180.8	201.2
impact energy (m J)	36.87	45.16	54.33	66.15	77.97	88.20	93.85	93.67	98.60
characteristic friction force F_{X0} (kg)	17	17	14	13	11	10	5	3	2
F_0/F_{X0}	1.12	1.12	1.36	1.46	1.73	1.90	3.81	6.35	9.52

3.3 Dynamic Nonlinear Problems in Rock

The cohesive force, internal friction, expansion and so on are typical characteristics of rock.
The rock has a capability to store energy on the microscopic level, which can return to macroscopic level under a given condition. This property can be regarded as one of fundamental features of rock. The rock can act as an energy contained media.

3.3 Dynamic Nonlinear Problems in Rock

When the rock is subjected to cycle of loading and unloading, the reversible elastic energy is far less than external force work of loading, a part of irreversible deformation energy dissipated and other parts of it accumulated in the rock. After unloading of external force, rock mass is be in “unstable” equilibrium state, namely, a small disturbance can make rock to be deformed continuously, and the deformation either may be with slow and progressive property of creep deformation, or with dynamic property of discontinuity. Here the released energy is far more than that of the small disturbance, which only take the effect of “initiating trigger”.

3.3 Dynamic Nonlinear Problems in Rock

（4） problem researched in the future

measurement of stability
(magnitude of contained energy, speed of release)
dynamic calculation of rock mass
mechanism of pendulum waves

Thank You All